Probing Lorentz violation with Doppler-shift experiments

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This work analyzes Doppler-effect experiments in terms of a general framework for violations of Lorentz symmetry: the Standard-Model Extension. These experiments are found to be sensitive to heretofore unprobed combinations of Lorentz-violation coefficients associated with protons and electrons. New bounds at the level of 10^{-11} and 10^{-8} for proton coefficients and 10^{-2} for electron coefficients emerge from a recent experiment.

I. INTRODUCTION

The Doppler effect of light is one of the classic tests of special relativity [1], particularly symmetry under Lorentz transformations. Modern experiments agree with relativistic predictions to a very high degree of precision [2]. However, the possibility of Lorentz symmetry violation due to Planck-scale physics [3, 4] makes continuing study of the relativistic Doppler shift an important endeavor.

This work uses the nongravitational sector of the Standard-Model Extension (SME), a general framework for violations of particle Lorentz symmetry, to analyze Doppler-effect experiments. Most previous analyses have used a limited test model [5] that can describe only a single type of matter aside from the photon, and that is likely a special case of the SME [6]. Analysis of the most recent Doppler experiment [2] has been performed [7] within the context of the photon sector of the SME, describing sensitivity to a combination of photon parameters; the photon sector is neglected in most of the current paper. Analysis within the context of the fermion sector of the SME has yet to be performed. Herein is presented such an analysis, yielding new bounds on a set of proton- and electron-associated coefficients for Lorentz violation. This work also indicates modifications to current experiments that could yield sensitivity to a much broader class of coefficients.

Theoretical analyses of the SME include its possible origin in spontaneously broken Lorentz symmetry [8], its definition [9], its spin-statistics relation and microcausal structure [10], its renormalizability [11], its extension to gravitational physics [12], and its hamiltonian for free Dirac fermions [13, 14]. Phenomenological studies of the SME include analyses of the electron sector [15, 16], atomic clocks [17–19], electrodynamics [6, 16, 20, 21], muon behavior [22], neutral mesons [23], neutrinos [24], the Higgs field [25], and more. Most of these phenomenological studies have been applied to existing experiments, including those with the electron sector [26], atomic clocks [27], electrodynamics [28], muon

behavior [29], neutral mesons [30], and neutrinos [31].

This paper is organized as follows. Section II collects many small discussions: the ingredients of a Doppler-effect experiment, qualitative reasons for expecting such experiments to be sensitive to Lorentz violation, definition of the key experimental quantity of interest, and relevant coordinate systems for detailed analysis. Section III describes the signals that would appear in Doppler experiments were Lorentz symmetry to be violated in accordance with the SME. The general results are applied to the completed Heidelberg experiment in Sec. IV. Section V discusses a few ways that future experiments could yield sensitivity to a broader set of Lorentz-violation coefficients. A summary appears in Sec. VI, and an Appendix contains some calculational details.

II. BASICS

Observer Lorentz symmetry is maintained in the SME, and hence conventional relationships hold between measurements made in different inertial frames of the same physical quantity. However, due to violation of particle Lorentz symmetry, analysis of experimental conditions has an extra subtlety. To wit: Since measurements made in identical experimental setups moving with respect to each other do *not* measure the same physical quantity, but rather different instances of similar physical quantities, they may not obey conventional relationships. For example, the rest-frame frequency associated with an atomic transition depends on the particular frame of the atom, in contrast to the conventional Lorentz-symmetric case.

A. Experimental basics

Doppler-shift experiments involve three main ingredients. The first ingredient consists of two groups of atoms, one group at rest in the lab frame and the other forming a beam with velocity $\vec{\beta}$ in the lab frame. The same transition should be studied in each group. A pair of coordinate frames, the lab frame $\mathcal{S}$ and the moving atoms' rest frame $\bar{\mathcal{S}}$, form the second ingredient. The third ingredient is a

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set of three transition-frequency measurements:

ν_{lab}= frequency associated with the atoms at rest in the lab frame, measured in the lab frame (and hence in their own rest frame),

$\bar{\nu}_{\mathrm{beam}}$= frequency associated with the beam atoms, measured in the beam frame (and hence in their own rest frame), and

ν_{beam}= frequency associated with the beam atoms, measured in the lab frame (*not* their own rest frame).

General considerations of Lorentz symmetry give qualitative insight into relationships within each of two pairs of the measurements. First, consider ν_{beam} and $\bar{\nu}_{\mathrm{beam}}$. These are measurements of the same transition in the same atoms, but measured in different frames, so they are related by an *observer* Lorentz transformation. Since the SME preserves observer Lorentz symmetry, ν_{beam} and $\bar{\nu}_{\mathrm{beam}}$ obey their conventional Doppler relationship in this theory.

Next, consider ν_{lab} and $\bar{\nu}_{\mathrm{beam}}$. These are not measurements of the same quantity. However, since they are measurements of the same type of thing (a particular transition in a particular type of atom), they are related by a *particle* Lorentz transformation [32]. Since the SME breaks particle Lorentz symmetry, they do not obey their conventional relationship, namely, equality. Doppler-effect experiments effectively probe the ratio of these two frequency measurements, bounding the amount by which it differs from unity.

In practice, ν_{beam} is not a single frequency measurement. Two different beam-atom transition frequencies are measured in the lab frame: ν_p, the frequency of light measured by a laser running parallel to the beam in the lab frame, and ν_a, the frequency of light measured by a laser running antiparallel to the beam in the lab frame. In conventional physics, a nice identity relates these to the beam transition frequency as measured in the beam frame: $\nu_a\nu_p = \bar{\nu}_{\mathrm{beam}}^2$. This relationship is based only on observer Lorentz transformations, and therefore holds in both conventional physics and the SME. Because of this, the analysis may be performed as if the frequencies $\bar{\nu}_{\mathrm{beam}}$ and ν_{lab} are directly measured, even though the actual experiments deal with frequencies ν_a, ν_p, and ν_{lab}.

The key experimental quantity is the ratio $\nu_a\nu_p/\nu_{\mathrm{lab}}^2$. Due to the identity mentioned above, this can be expressed as the square of the ratio of two transition frequencies as measured in their own rest frames, $\bar{\nu}_{\mathrm{beam}}$ and ν_{lab}. As discussed above, these frequencies are equal in conventional physics, but may be different whenever particle Lorentz symmetry is violated. Thus, in conventional physics, the experimental quantity of interest is exactly unity. When Lorentz symmetry is violated, however, it may differ slightly from one. Moreover, for experiments conducted in a frame that is not strictly inertial (such as those based on Earth's surface), the key ratio may vary with time.

Since particle Lorentz symmetry is at least approximately valid, the experimental quantity of interest can conveniently be written as a perturbation around one:

$$\frac{\nu_a\nu_p}{\nu_{\mathrm{lab}}^2} = \frac{\bar{\nu}_{\mathrm{beam}}^2}{\nu_{\mathrm{lab}}^2} = 1 + \varepsilon \quad , \qquad (1)$$

where $\varepsilon \ll 1$. Section III of this paper describes the size and time dependence of ε under general experimental scenarios.

The most stringent bound on ε to date results from a recent Doppler-effect experiment at the heavy-ion storage ring TSR in Heidelberg [2]. This experiment limits the magnitude of ε to

$$|\varepsilon| \lesssim 2 \times 10^{-9} \quad . \qquad (2)$$

B. Coordinate frames.

Three coordinate frames are necessary for this work. The first is a nonrotating system (T, X, Y, Z) centered on the Sun. This frame is approximately inertial over the course of any realistic Doppler-shift experiment. It is defined in detail in Section III of Ref. [19].

The second system (t, x, y, z) is fixed in the laboratory on Earth's surface. In it, the x axis points south, the y axis points east, and the z axis points vertically upwards. Useful angles in this frame include the angle $\eta \approx 23.4°$ between Earth's equatorial and orbital planes and the colatitude χ of the laboratory. The angular frequency of Earth's revolution about the Sun is denoted $\Omega_\oplus$, while the rotational frequency of the Earth is denoted $\omega_\oplus$. The current work uses $\beta_\oplus \approx 10^{-4}$ to denote the average speed of Earth with respect to the Sun, and $\beta_L \approx (1.5 \times 10^{-6})\sin\chi$ to denote the average speed of the lab with respect to Earth's rotational axis. These first two systems and transformations between them are defined in detail in Appendix C of Ref. [6].

The third frame $(\bar{t}, \bar{x}, \bar{y}, \bar{z})$ is the rest frame of the beam atoms. These axes are defined to move with speed β along the lab-frame y-axis. The $\bar{y}$ axis is chosen to be parallel to the y axis. The $\bar{z}$ axis points along the quantization axis of the beam atoms, which is assumed to be rotated around the y axis by angle θ away from the z axis. Thus, for an experiment conducted at a colatitude χ, the angle between Earth's rotation axis and the quantization axis is given by $\xi = \chi + \theta$. The current work doesn't include generalization to beams moving with arbitrary orientation: such generalization is straightforward and has little effect on the analysis aside from computational messiness. The $\bar{x}$ axis is defined so as to complete the right-handed system.

This work keeps only terms up to second order in β for two reasons: First, for all existing Doppler-effect experiments to date, $\beta \ll 1$. In the most recent experiment [2], for example, $\beta \approx 0.064$. Second, the types of Lorentz violation to which Doppler-effect experiments are sensitive

are unaffected by inclusion of higher-order terms. Moreover, at most first-order terms in $\beta_\oplus$ and β_L are kept since each is much smaller than β.

III. SME PREDICTIONS

In this section, the behavior of the key experimental quantity is described as predicted by the SME. First, the shift to an atom's transition frequency is considered in an approximately inertial frame instantaneously at rest with the atom. Next, the frequencies of atoms moving at constant velocity with respect to each other are compared. The section concludes with the consequences of the motion of the atoms as they rotate with Earth and ride Earth around the Sun.

A. Approximately inertial frame

In the SME, new interactions cause shifts in atomic energy levels with respect to conventional values [17]. The energy-level shift δE to an atomic state with total-angular-momentum quantum number F and projection number m_F takes the form $\delta E(F, m_F) = \widehat{m}_F E_d(F) + \widetilde{m}_F E_q(F)$. In this expression, $\widehat{m}_F$ and $\widetilde{m}_F$ are ratios of Clebsch-Gordan coefficients while $E_d(F)$ and $E_q(F)$ are m_F-independent dipole- and quadrupole-type energy-level shifts.

In turn, this generally induces transition-frequency shifts of the form

$$\nu = \nu_{\text{SM}} + \delta\nu \quad,$$
$$\delta\nu = \frac{1}{2\pi} \sum_w \left[(\widehat{m}_F \beta_w - \widehat{m}'_F \beta'_w) \tilde{b}_3^w \right.$$
$$+ (\widehat{m}_F \delta_w - \widehat{m}'_F \delta'_w) \tilde{d}_3^w$$
$$+ (\widehat{m}_F \kappa_w - \widehat{m}'_F \kappa'_w) \tilde{g}_d^w$$
$$+ (\widetilde{m}_F \gamma_w - \widetilde{m}'_F \gamma'_w) \tilde{c}_q^w$$
$$\left. + (\widetilde{m}_F \lambda_w - \widetilde{m}'_F \lambda'_w) \tilde{g}_q^w \right] \quad, \qquad (3)$$

where ν denotes any transition frequency measured in the atom's own rest frame, assuming that the tensor components are evaluated in that frame; ν_{SM} is the conventional (i.e., according-to-the-standard-model) frequency; and $\delta\nu$ is the shift induced by the SME. The sum is over the particle types proton, neutron, and electron. The quantities $\beta_w, \dots, \lambda_w$ that describe m_F-independent properties of the atoms are described in the paragraphs surrounding Eqs. (10) through (12) of Ref. [17]. Finally, $\tilde{b}_3^w, \dots, \tilde{g}_q^w$ are the specific combinations of Lorentz-violation tensor components that appear in studies of atomic systems; explicit definitions of them appear as Eq. (9) of Ref. [17].

B. Comparison of lab and beam atoms

Expression (3) holds for both ν_{lab} and $\bar{\nu}_{\text{beam}}$, with the tensor components evaluated in the lab and beam frames, respectively. Since tensor components generally depend on the frame in which they're evaluated, it usually pertains that $\tilde{b}_{\bar{3}} \neq \tilde{b}_3$, $\tilde{c}_{\bar{q}} \neq \tilde{c}_q$, etc. Thus, since they depend on tensor components evaluated in different frames, $\delta\bar{\nu}_{\text{beam}} \neq \delta\nu_{\text{lab}}$ and hence $\bar{\nu}_{\text{beam}} \neq \nu_{\text{lab}}$.

Since Doppler-effect experiments involve the same transitions in the same atomic species (albeit measured in different frames), the values of $\widehat{m}_F$, $\widetilde{m}_F$, and $\beta_w, \dots,$ λ_w that appear in $\delta\bar{\nu}_{\text{beam}}$ and $\delta\nu_{\text{lab}}$ are identical. Thus, the experimental quantity of interest becomes

$$\frac{\bar{\nu}_{\text{beam}}^2}{\nu_{\text{lab}}^2} = 1 + \frac{2}{\nu_{\text{SM}}} (\delta\bar{\nu}_{\text{beam}} - \delta\nu_{\text{lab}})$$
$$= 1 + \frac{1}{\pi\nu_{\text{SM}}} \sum_w \left[(\widehat{m}_F \beta_w - \widehat{m}'_F \beta'_w)(\tilde{b}_{\bar{3}} - \tilde{b}_3) \right.$$
$$+ (\widehat{m}_F \delta_w - \widehat{m}'_F \delta'_w)(\tilde{d}_{\bar{3}} - \tilde{d}_3)$$
$$+ (\widehat{m}_F \kappa_w - \widehat{m}'_F \kappa'_w)(\tilde{g}_{\bar{d}} - \tilde{g}_d)$$
$$+ (\widetilde{m}_F \gamma_w - \widetilde{m}'_F \gamma'_w)(\tilde{c}_{\bar{q}} - \tilde{c}_q)$$
$$\left. + (\widetilde{m}_F \lambda_w - \widetilde{m}'_F \lambda'_w)(\tilde{g}_{\bar{q}} - \tilde{g}_q) \right] \quad. \quad (4)$$

This quantity varies with time as Earth rotates. Its exact time dependence is calculated in Section III C.

Some of the coefficients that appear in Eq. (4) are given in the lab frame, while others are given in comoving beam coordinates. Conventional Lorentz boosts may be used to express the beam components in terms of the lab-frame components. To second order in the beam speed β and expressed entirely in lab-frame coordinates, the combinations of coefficients in Eq. (4) are

$$\tilde{b}_{\bar{3}} - \tilde{b}_3 = \beta \left[-md_{32} + mg_{122} + mg_{100} - H_{10} \right]$$
$$+ \beta^2 \left[-\tfrac{1}{2}md_{30} + mg_{120} + mg_{102} - \tfrac{1}{2}H_{12} \right] \quad,$$
$$\tilde{d}_{\bar{3}} - \tilde{d}_3 = \beta \left[md_{23} + \tfrac{1}{2}md_{32} - \tfrac{1}{2}H_{10} \right]$$
$$+ \beta^2 \left[\tfrac{1}{4}md_{30} + \tfrac{1}{2}md_{03} - \tfrac{1}{4}H_{12} \right] \quad,$$
$$\tilde{g}_{\bar{d}} - \tilde{g}_d = \beta \left[2mg_{122} + 2mg_{100} \right]$$
$$+ \beta^2 \left[2mg_{120} + 2mg_{102} \right] \quad,$$
$$\tilde{c}_{\bar{q}} - \tilde{c}_q = \beta \left[mc_{20} + mc_{02} \right]$$
$$+ \beta^2 \left[mc_{11} + 2mc_{22} + mc_{33} \right] \quad, \quad \text{and}$$
$$\tilde{g}_{\bar{q}} - \tilde{g}_q = \beta \left[-mg_{211} + 2mg_{233} + mg_{200} \right]$$
$$+ \beta^2 \left[\tfrac{1}{2}mg_{101} + \tfrac{1}{2}mg_{202} - mg_{303} \right] \quad. \quad (5)$$

C. Noninertial frame

As the coefficients that appear in Eq. (5) are tensor components in a frame attached to Earth's surface, they vary as Earth rotates and accelerates through the nonrotating Sun frame. Thus, it is best to re-express the key experimental quantity in terms of tensor components

in the Sun frame, thereby explicitly displaying its time dependence. In this section, $\tilde{c}_{\bar{q}} - \tilde{c}_q$ is first discussed in detail, then relevant information for all coefficients is summarized.

In terms of nonrotating-frame components,

$$
\begin{aligned}
\tilde{c}_{\bar{q}} - \tilde{c}_q = \; & \cos\omega_\oplus T_\oplus \Big\{ \beta\Big[\tilde{c}_{TY}\Big] + \beta\beta_\oplus\Big[c_\eta c_{\Omega T}\tilde{c}_- - \tfrac{1}{3}c_\eta c_{\Omega T}\tilde{c}_Q - s_\eta c_{\Omega T}\tilde{c}_X + s_{\Omega T}\tilde{c}_Z\Big] + \beta^2\beta_L\Big[2\tilde{c}_{TY}\Big]\Big\} \\
& + \sin\omega_\oplus T_\oplus \Big\{ \beta\Big[-\tilde{c}_{TX}\Big] + \beta\beta_\oplus\Big[-s_{\Omega T}\tilde{c}_- - \tfrac{1}{3}s_{\Omega T}\tilde{c}_Q + s_\eta c_{\Omega T}\tilde{c}_Y + c_\eta c_{\Omega T}\tilde{c}_Z\Big] + \beta^2\beta_L\Big[-2\tilde{c}_{TX}\Big]\Big\} \\
& + \cos 2\omega_\oplus T_\oplus \Big\{ \beta^2\Big[-\tfrac{1}{2}\tilde{c}_-\Big] + \beta\beta_L\Big[-\tilde{c}_-\Big] + \beta^2\beta_\oplus\Big[-\tfrac{1}{2}s_{\Omega T}\tilde{c}_{TX} - \tfrac{1}{2}c_\eta c_{\Omega T}\tilde{c}_{TY}\Big]\Big\} \\
& + \sin 2\omega_\oplus T_\oplus \Big\{ \beta^2\Big[-\tfrac{1}{2}\tilde{c}_Z\Big] + \beta\beta_L\Big[-\tilde{c}_Z\Big] + \beta^2\beta_\oplus\Big[\tfrac{1}{2}c_\eta c_{\Omega T}\tilde{c}_{TX} - \tfrac{1}{2}s_{\Omega T}\tilde{c}_{TY}\Big]\Big\} \\
& + \Big\{ \beta^2\Big[\tfrac{1}{6}\tilde{c}_Q\Big] + \beta\beta_L\Big[\tfrac{1}{3}\tilde{c}_Q\Big] + \beta^2\beta_\oplus\Big[\tfrac{3}{2}s_{\Omega T}\tilde{c}_{TX} - \tfrac{3}{2}c_\eta c_{\Omega T}\tilde{c}_{TY} - s_\eta c_{\Omega T}\tilde{c}_{TZ}\Big]\Big\} \quad .
\end{aligned} \tag{6}
$$

	$\omega_\oplus T_\oplus$ dep.	$\tilde{c}_Q$	$\tilde{c}_X$	$\tilde{c}_Y$	$\tilde{c}_Z$	$\tilde{c}_-$	$\tilde{c}_{TX}$	$\tilde{c}_{TY}$	$\tilde{c}_{TZ}$
$\tilde{c}_{\bar{q}} - \tilde{c}_q$	$\cos\omega_\oplus T_\oplus$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	β	$\cdot$
	$\sin\omega_\oplus T_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	β	$\cdot$	$\cdot$
	$\cos 2\omega_\oplus T_\oplus$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	β^2	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\cdot$
	$\sin 2\omega_\oplus T_\oplus$	$\cdot$	$\cdot$	$\cdot$	β^2	$\cdot$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\cdot$
	const.	β^2	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$

TABLE I: Dependance of $\tilde{c}_{\bar{q}} - \tilde{c}_q$ on the lab's rotational frequency $\omega_\oplus$ as Earth rotates, on Sun-frame tilde coefficients, and on various speeds.

In this and later equations, the abbreviations $c_x := \cos x$ and $s_x := \sin x$ are used. Further, the combination $\Omega_\oplus T$ is shortened to ΩT. The individual terms in this expression have been sorted into five pieces based on their variation at Earth's relatively rapid rotational frequency $\omega_\oplus$:

$$
\begin{aligned}
\tilde{c}_{\bar{q}} - \tilde{c}_q = \; & C_1 \cos\omega_\oplus T_\oplus + S_1 \sin\omega_\oplus T_\oplus \\
& + C_2 \cos 2\omega_\oplus T_\oplus + S_2 \sin 2\omega_\oplus T_\oplus + K_0 \quad , \tag{7}
\end{aligned}
$$

where the coefficients of these sinusoids and the piece K_0 include both terms that vary at the relatively slow frequency $\Omega_\oplus$ of Earth's motion around the Sun and terms that are truly constant. All combinations of Sun-frame components $\tilde{c}_Q$, $\tilde{c}_{TX}$, etc. in this and later expressions are defined in Appendix B of [19].

The relevant content of Eq. (6) deals with the $\omega_\oplus$ time dependence, the set of nonrotating-frame parameters on which it depends, and the suppression of each of these parameters by products of various speeds. Exact dependence on order-one factors such as sines and cosines of ξ or η only changes sensitivity by factors of order unity, and is therefore relatively unimportant.

The relevant content is summarized in Table I. Each column corresponds to a different Sun-frame tilde combination of coefficients for Lorentz violation, while each row corresponds to a type of time dependence. The entries in the table give the suppression factor for the dominant contribution of each Sun-frame tilde combination to each type of time dependence. For example, the $-\tfrac{1}{3}\beta\beta_\oplus c_\eta c_{\Omega T}\tilde{c}_Q \cos\omega_\oplus T_\oplus$ term in Eq. (6) is represented by the upper-left $\beta\beta_\oplus$ entry in Table I. Note that the $\tfrac{1}{3}\beta\beta_L\tilde{c}_Q$ constant term is not represented in the table, as it is dominated by the $\tfrac{1}{6}\beta^2\tilde{c}_Q$ term.

Detailed expressions like Eq. (6) for other coefficients are unwieldy and give no particular insight. The relevant information is displayed two ways. First, the time and Sun-frame-component dependence is summarized in Tables II and III, which are entirely analogous to Table I.

Second, the detailed results in the limits $\beta_L \to 0$, $\beta_\oplus \to 0$ are given. This corresponds to neglecting relativistic effects associated with the lab's motion through space. It is equivalent to assuming that the lab has a constant velocity, but rotates about an axis parallel to Earth's. Since $\beta_L \ll \beta$ and $\beta_\oplus \ll \beta$ in any current experiment, all highest-order effects are represented. The coefficient differences listed in Eq. (5), when expressed in Sun-frame components, are given in the Appendix.

IV. THE HEIDELBERG HEAVY-ION EXPERIMENT

The recent experiment at the Heidelberg heavy-ion storage facility [2] used saturation spectroscopy within $^7\text{Li}^+$ ions moving at speed $\beta \approx 0.064$ to probe the Doppler effect. The experiment effectively averaged over all of the $\Delta m_F = 0$ transitions between the $^3P_2(F = 7/2)$ and $^3S_1(F = 5/2)$ levels, to each of which the analysis of Sec. III may be applied. Thus, the measured frequency

	$\omega_\oplus T_\oplus$ dep.	$\tilde{b}_X$	$\tilde{b}_Y$	$\tilde{b}_Z$	$\tilde{d}_X$	$\tilde{d}_Y$	$\tilde{d}_Z$	$\tilde{g}_{TX}$	$\tilde{g}_{TY}$	$\tilde{g}_{TZ}$	$\tilde{g}_-$	$\tilde{g}_{DX}$	$\tilde{g}_{DY}$	$\tilde{g}_{DZ}$	$\tilde{g}_Q$	$\tilde{H}_{XT}$	$\tilde{H}_{YT}$	$\tilde{H}_{ZT}$
$\tilde{b}_{\bar{3}} - \tilde{b}_3$	$\cos\omega_\oplus T_\oplus$	β^2	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\beta\beta_\oplus$	$\cdot$	β^2	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	β^2	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	β	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$
	$\sin\omega_\oplus T_\oplus$	$\beta\beta_\oplus$	β^2	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\beta\beta_\oplus$	β^2	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	β^2	$\beta\beta_\oplus$	$\cdot$	$\beta^2\beta_\oplus$	β	$\beta^2\beta_\oplus$
	$\cos 2\omega_\oplus T_\oplus$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	β^2	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$
	$\sin 2\omega_\oplus T_\oplus$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	β^2	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$
	const.	$\beta\beta_\oplus$	$\beta\beta_\oplus$	β^2	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	β^2	$\beta\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	β
$\tilde{d}_{\bar{3}} - \tilde{d}_3$	$\cos\omega_\oplus T_\oplus$	$\cdot$	$\beta\beta_\oplus$	$\cdot$	β^2	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	β	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$
	$\sin\omega_\oplus T_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\beta\beta_\oplus$	β^2	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\beta^2\beta_\oplus$	β	$\beta^2\beta_\oplus$
	$\cos 2\omega_\oplus T_\oplus$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$
	$\sin 2\omega_\oplus T_\oplus$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$
	const.	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	β^2	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	β
$\tilde{g}_{\bar{d}} - \tilde{g}_d$	$\cos\omega_\oplus T_\oplus$	β^2	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\cdot$	β^2	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	β^2	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\cdot$	$\cdot$
	$\sin\omega_\oplus T_\oplus$	$\beta\beta_\oplus$	β^2	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\cdot$	$\beta\beta_\oplus$	β^2	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	β^2	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\cdot$	$\cdot$
	$\cos 2\omega_\oplus T_\oplus$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	β^2	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$
	$\sin 2\omega_\oplus T_\oplus$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	β^2	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$
	const.	$\beta\beta_\oplus$	$\beta\beta_\oplus$	β^2	$\cdot$	$\cdot$	$\cdot$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	β^2	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\cdot$
$\tilde{g}_{\bar{q}} - \tilde{g}_q$	$\cos\omega_\oplus T_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\cdot$	$\beta\beta_\oplus$	β^2	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\cdot$
	$\sin\omega_\oplus T_\oplus$	$\cdot$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\cdot$	β^2	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\cdot$
	$\cos 2\omega_\oplus T_\oplus$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	β^2	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$
	$\sin 2\omega_\oplus T_\oplus$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	β^2	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$
	const.	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\cdot$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\cdot$	$\cdot$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	$\beta\beta_\oplus$	β^2	$\cdot$	$\cdot$	$\cdot$

TABLE II: Dependance of $\tilde{b}_{\bar{3}} - \tilde{b}_3$, $\tilde{d}_{\bar{3}} - \tilde{d}_3$, $\tilde{g}_{\bar{d}} - \tilde{g}_d$, $\tilde{g}_{\bar{q}} - \tilde{g}_q$ on the lab's rotational frequency $\omega_\oplus$ as Earth rotates, on Sun-frame tilde coefficients, and on various speeds (continued in Table III).

shift in each group of atoms is

$$\langle \delta\nu \rangle = \frac{1}{6} \sum_{m_F=-5/2}^{+5/2} \left[\frac{\delta E(F=\frac{7}{2}, m_F) - \delta E(F=\frac{5}{2}, m_F)}{2\pi} \right]$$

$$= \frac{1}{12\pi} \sum_{m_F=-5/2}^{+5/2} \left[\widehat{m}_F E_d(F=\tfrac{7}{2}) + \widetilde{m}_F E_q(F=\tfrac{7}{2}) \right.$$
$$\left. - \widehat{m}'_F E_d(F=\tfrac{5}{2}) - \widetilde{m}'_F E_q(F=\tfrac{5}{2}) \right] \quad . \quad (8)$$

Since $\widehat{m}_F$ is linear in m_F, all E_d contributions to this sum cancel each other out. Using the fact that any sum of $\widehat{m}_F$ or $\widetilde{m}_F$ over a complete set of $m_F = -F, \cdots, +F$ is zero [17], the $F = \frac{5}{2}$ parts of this sum contribute nothing. Finally, the $E_q(F = \frac{7}{2})$ sum can be explicitly tallied to give

$$\langle \delta\nu \rangle = -\frac{1}{6\pi} E_q(F=\tfrac{7}{2}) = -\frac{1}{6\pi} \sum_w \left(\gamma_w \tilde{c}_q^w + \lambda_w \tilde{g}_q^w \right) \quad . \quad (9)$$

The key experimental quantity for the Heidelberg experiment is then

$$\frac{\bar{\nu}_{\text{beam}}^2}{\nu_{\text{lab}}^2} = 1 - \frac{1}{3\pi\nu_{\text{SM}}} \sum_w \left[\gamma_w(\tilde{c}_{\bar{q}}^w - \tilde{c}_q^w) + \lambda_w(\tilde{g}_{\bar{q}}^w - \tilde{g}_q^w) \right] \quad . \quad (10)$$

Though calculation of the γ_w and λ_w parameters for nucleons may in general be quite involved [17], the nuclear Schmidt model suffices for this work. For ^{7}Li$^+$ in the $F = \frac{7}{2}$ state, the Schmidt model gives $\gamma_p \approx -\frac{1}{15} \times 10^{-2}$ and $\gamma_n = 0$, while standard atomic calculations lead to $\gamma_e \approx -\frac{1}{15} \times 10^{-5}$. For each particle species, Schmidt model and standard atomic calculations give $\lambda_w = 0$, so $\tilde{g}_{\bar{q}} - \tilde{g}_q$ makes no contribution.

An expression for $\tilde{c}_{\bar{q}} - \tilde{c}_q$ in terms of nonrotating-frame components is given by Eq. (6). The Heidelberg experiment effectively averaged frequency measurements over several days [33], so the contributions proportional to sines and cosines involving sidereal frequency $\omega_\oplus$ all average to approximately zero [34]. Hence, the effective value of $\tilde{c}_{\bar{q}} - \tilde{c}_q$ is

$$\tilde{c}_{\bar{q}} - \tilde{c}_q = \beta^2\beta_\oplus \left(\tfrac{3}{2} s_{\Omega T} \tilde{c}_{TX} - \tfrac{3}{2} c_\eta c_{\Omega T} \tilde{c}_{TY} - s_\eta c_{\Omega T} \tilde{c}_{TZ} \right)$$
$$+ \beta^2 \left(\tfrac{1}{6} \tilde{c}_Q \right) + \beta\beta_L \left(\tfrac{1}{3} \tilde{c}_Q \right) \quad . \quad (11)$$

	$\omega_\oplus T_\oplus$ dep.	$\tilde{d}_{XY}$	$\tilde{d}_{YZ}$	$\tilde{d}_{ZX}$	$\tilde{d}_Q$	$\tilde{d}_+$	$\tilde{d}_-$	$\tilde{b}_T$	$\tilde{g}_T$	$\tilde{g}_c$	$\tilde{g}_{XY}$	$\tilde{g}_{XZ}$	$\tilde{g}_{YX}$	$\tilde{g}_{YZ}$	$\tilde{g}_{ZX}$	$\tilde{g}_{ZY}$
$\tilde{b}_{\bar{3}} - \tilde{b}_3$	$\cos\omega_\oplus T_\oplus$	$\beta^2\beta_\oplus$	·	·	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta\beta_L$	·	·	$\beta^2\beta_\oplus$	·	$\beta^2\beta_\oplus$
	$\sin\omega_\oplus T_\oplus$	·	$\beta^2\beta_\oplus$	β	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	·	$\beta^2\beta_\oplus$	$\beta\beta_L$	·	$\beta^2\beta_\oplus$	·
	$\cos 2\omega_\oplus T_\oplus$	β	·	·	·	·	·	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	·	$\beta^2\beta_\oplus$	·	$\beta\beta_L$	$\beta\beta_L$
	$\sin 2\omega_\oplus T_\oplus$	·	·	·	·	·	β	β	β	β	$\beta^2\beta_\oplus$	·	$\beta^2\beta_\oplus$	·	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$
	const.	β	·	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	·	·	·	·	$\beta^2\beta_\oplus$	·	$\beta^2\beta_\oplus$	·	$\beta\beta_L$	$\beta\beta_L$
$\tilde{d}_{\bar{3}} - \tilde{d}_3$	$\cos\omega_\oplus T_\oplus$	$\beta^2\beta_\oplus$	β	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	·	β	β	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$
	$\sin\omega_\oplus T_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	β	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	·	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	β	β	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$
	$\cos 2\omega_\oplus T_\oplus$	β	·	·	·	·	·	·	·	·	·	·	·	·	·	β	β
	$\sin 2\omega_\oplus T_\oplus$	·	·	·	·	·	β	·	·	·	·	·	·	·	·	·	·
	const.	β	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	·	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	·	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	β	β
$\tilde{g}_{\bar{d}} - \tilde{g}_d$	$\cos\omega_\oplus T_\oplus$	·	·	·	·	·	·	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	β	·	·	$\beta^2\beta_\oplus$	·	$\beta^2\beta_\oplus$
	$\sin\omega_\oplus T_\oplus$	·	·	·	·	·	·	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	·	$\beta^2\beta_\oplus$	β	·	$\beta^2\beta_\oplus$	·
	$\cos 2\omega_\oplus T_\oplus$	·	·	·	·	·	·	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	·	$\beta^2\beta_\oplus$	·	β	β
	$\sin 2\omega_\oplus T_\oplus$	·	·	·	·	·	·	β	β	β	$\beta^2\beta_\oplus$	·	$\beta^2\beta_\oplus$	·	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$
	const.	·	·	·	·	·	·	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	·	$\beta^2\beta_\oplus$	·	$\beta^2\beta_\oplus$	·	β	β
$\tilde{g}_{\bar{q}} - \tilde{g}_q$	$\cos\omega_\oplus T_\oplus$	·	·	·	·	·	·	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	·	$\beta^2\beta_\oplus$	β	β	$\beta^2\beta_\oplus$	·
	$\sin\omega_\oplus T_\oplus$	·	·	·	·	·	·	·	·	$\beta^2\beta_\oplus$	β	β	·	$\beta^2\beta_\oplus$	·	$\beta^2\beta_\oplus$
	$\cos 2\omega_\oplus T_\oplus$	·	·	·	·	·	·	β	β	β	$\beta^2\beta_\oplus$	·	$\beta^2\beta_\oplus$	·	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$
	$\sin 2\omega_\oplus T_\oplus$	·	·	·	·	·	·	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	·	$\beta^2\beta_\oplus$	·	β	β
	const.	·	·	·	·	·	·	β	β	·	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$	$\beta^2\beta_\oplus$

TABLE III: Dependance of $\tilde{b}_{\bar{3}} - \tilde{b}_3$, $\tilde{d}_{\bar{3}} - \tilde{d}_3$, $\tilde{g}_{\bar{d}} - \tilde{g}_d$, $\tilde{g}_{\bar{q}} - \tilde{g}_q$ on the lab's rotational frequency $\omega_\oplus$ as Earth rotates, on Sun-frame tilde coefficients, and on various speeds (continued from Table II).

Note that this result could be derived from Table I, though several order-one factors would be missing. Also missing would be the last term. However, since it is dominated by the next-to-last term, it is irrelevant. It is neglected in the following calculations.

The key quantity for the Heidelberg experiment is then

$$\frac{\bar{\nu}^2_{\text{beam}}}{\nu^2_{\text{lab}}} = 1 - \frac{1}{3\pi\nu_{\text{SM}}} \sum_{w=p,e} \gamma_w \left[\frac{1}{6}\beta^2 \tilde{c}^w_Q + \beta^2\beta_\oplus \left(\frac{3}{2}s_{\Omega T}\tilde{c}^w_{TX} - \frac{3}{2}c_\eta c_{\Omega T}\tilde{c}^w_{TY} - s_\eta c_{\Omega T}\tilde{c}^w_{TZ} \right) \right] (12)$$

Due to experimental data being taken over a small number of days, $c_{\Omega T}$ and $s_{\Omega T}$ are each approximately constants of order one, as are c_η and s_η. Comparison with Eq. (2) and expression of the coefficient combinations in terms of original SME coefficients (i.e., in terms of "nontilde" coefficients) leads to rough bounds

$$|c^p_{XX} + c^p_{YY} - 2c^p_{ZZ}| \lesssim 10^{-11} \quad,$$
$$|c^p_{TJ} + c^p_{JT}| \lesssim 10^{-8} \quad (J = X,Y,Z) \quad,$$
$$|c^e_{XX} + c^e_{YY} - 2c^e_{ZZ}| \lesssim 10^{-5} \quad, \text{ and}$$
$$|c^e_{TJ} + c^e_{JT}| \lesssim 10^{-2} \quad (J = X,Y,Z) \quad.(13)$$

Though detailed calculations have not been performed, more realistic nuclear models likely give similar-size bounds to many other parameters, including $\tilde{c}_Q$ for the neutron and $(\tilde{g}_{\bar{q}} - \tilde{g}_q)$-associated coefficients for both the proton and neutron.

Nearly all of these bounds are new — only $c_{XX} + c_{YY} - 2c_{ZZ}$ for the electron has been previously bounded. They complement recent bounds on certain electron c_{MN} combinations [16], spin-torsion and Penning-trap bounds on other electron coefficients [26], and dipole-type bounds on proton and electron coefficients derived from clock-comparison experiments [17, 27].

This work neglects effects from the photon sector of the SME. However, such effects [7] can be readily appended to this analysis. Under the assumptions and notation of this work, photon contributions to the key experimental quantity would appear as an additional term $4\beta\beta_\oplus s_\eta s_\xi c_{\Omega T}\tilde{\kappa}_{\text{tr}}$ in Eq. (12). This leads to the bound on $\tilde{\kappa}_{\text{tr}}$ quoted in the earlier work.

V. FUTURE EXPERIMENTS

There are two broad classes of changes that could be made to future Doppler-effect experiments to increase the set of coefficient sensitivity while using the same atomic species. The first involves study of a different set of transitions, while the second involves the time dependence of frequency measurements.

First, an experiment studying a different set of transitions could be sensitive to several dipole-type coefficient differences that appear in Eq. (4). The different transition set could be a reduced subset of $\Delta m_F = 0$ transitions, or a set of $\Delta m_F \neq 0$ transitions. Calculations like those that appear in Sec. IV of the current work may be used to determine the set of coefficient differences to which a given experiment is sensitive. For example, the standard clock transition $m_F = 0 \to m_F{}' = 0$ is incapable of measuring any Lorentz-violation effects.

Changes of the first type would induce the relevance of more rows of Tables II and III. Without incorporating the time-dependence change described below, only those associated with time-constant contributions would appear. This could lead to sensitivity to all Sun-frame combinations except $\tilde{g}_{TZ}$, $\tilde{g}_-$, $\tilde{g}_c$, and $\tilde{d}_-$ from Tables II and III and $\tilde{c}_X$, $\tilde{c}_Y$, $\tilde{c}_Z$, and $\tilde{c}_-$ from Table I. Sensitivity to these coefficient combinations requires implementation of the second change.

Second, timing information could be kept to allow study of the key experimental quantity as the lab rotates and accelerates through the Sun frame. This would lead to sensitivity to the sinusoidal rows of Table I. Combination of this change together with the first could lead to sensitivity to the sinusoidal rows of Tables II and III as well.

VI. SUMMARY

This work describes an analysis of Doppler-shift experiments in the context of Lorentz violation, conducted within the framework of the Standard-Model Extension. Included are effects up to second order in the beam atoms' speed with respect to the lab, first order in Earth's speed with respect to the Sun. and first order in the lab's speed with respect to Earth's rotational axis. The key experimental quantity is expressed in terms of combinations of nonrotating-Sun-frame Lorentz-violation coefficients, atomic/nuclear expectation values, geometrical factors, and time.

The most recent Doppler-shift experiment at the Heidelberg storage ring is studied in detail. It is found to yield constraints on several previously unprobed Lorentz-violation coefficients associated with protons at the level of 10^{-11} and 10^{-8}, and coefficients associated with electrons at the level of 10^{-2}. Bounds on many other coefficients for neutrons and protons would likely result from more detailed nuclear calculations.

APPENDIX: LAB-FRAME COEFFICIENT DIFFERENCES

Below are given the coefficient differences listed in Eq. (5) in the limits $\beta_\oplus \to 0$, $\beta_L \to 0$, when expressed in Sun-frame components.

$$
\begin{aligned}
\tilde{b}_{\bar{3}} - \tilde{b}_3 =\ & \cos\omega_\oplus T_\oplus \Big\{ \beta\Big[-c_\xi \tilde{H}_{XT}\Big] + \beta^2\Big[\tfrac{1}{2}s_\xi \tilde{b}_X + \tfrac{1}{2}s_\xi \tilde{g}_{DX} - \tfrac{1}{2}s_\xi \tilde{g}_{TX}\Big]\Big\} \\
+\ & \sin\omega_\oplus T_\oplus \Big\{ \beta\Big[c_\xi \tilde{d}_{ZX} - c_\xi \tilde{H}_{YT}\Big] + \beta^2\Big[\tfrac{1}{2}s_\xi \tilde{b}_Y + \tfrac{1}{2}s_\xi \tilde{g}_{DY} + \tfrac{1}{2}s_\xi \tilde{g}_{TY}\Big]\Big\} \\
+\ & \cos 2\omega_\oplus T_\oplus \Big\{ \beta\Big[-\tfrac{1}{2}s_\xi \tilde{d}_{XY}\Big] + \beta^2\Big[\tfrac{1}{2}c_\xi \tilde{g}_{TZ}\Big]\Big\} \\
+\ & \sin 2\omega_\oplus T_\oplus \Big\{ \beta\Big[\tfrac{1}{2}s_\xi \tilde{d}_- + \tfrac{1}{2}s_\xi \tilde{b}_T - s_\xi \tilde{g}_c - \tfrac{1}{2}s_\xi \tilde{g}_T\Big] + \beta^2\Big[-\tfrac{1}{2}c_\xi \tilde{g}_-\Big]\Big\} \\
+\ & \Big\{ \beta\Big[-\tfrac{1}{2}s_\xi \tilde{d}_{XY} + s_\xi \tilde{H}_{ZT}\Big] + \beta^2\Big[\tfrac{1}{2}c_\xi \tilde{b}_Z + \tfrac{1}{2}c_\xi \tilde{g}_{DZ}\Big]\Big\} \quad,
\end{aligned} \tag{A.1}
$$

$$
\begin{aligned}
\tilde{d}_{\bar{3}} - \tilde{d}_3 =\ & \cos\omega_\oplus T_\oplus \Big\{ \beta\Big[c_\xi \tilde{d}_{YZ} + \tfrac{1}{2}c_\xi \tilde{g}_{XY} - c_\xi \tilde{g}_{XZ} - \tfrac{1}{2}c_\xi \tilde{H}_{XT}\Big] + \beta^2\Big[\tfrac{1}{2}s_\xi \tilde{d}_X\Big]\Big\} \\
+\ & \sin\omega_\oplus T_\oplus \Big\{ \beta\Big[-\tfrac{1}{2}c_\xi \tilde{d}_{ZX} + \tfrac{1}{2}c_\xi \tilde{g}_{YX} - c_\xi \tilde{g}_{YZ} - \tfrac{1}{2}c_\xi \tilde{H}_{YT}\Big] + \beta^2\Big[\tfrac{1}{2}s_\xi \tilde{d}_Z\Big]\Big\} \\
+\ & \cos 2\omega_\oplus T_\oplus \Big\{ \beta\Big[\tfrac{3}{4}s_\xi \tilde{d}_{XY} + \tfrac{3}{4}s_\xi \tilde{g}_{ZX} - \tfrac{3}{4}s_\xi \tilde{g}_{ZY}\Big]\Big\} \\
+\ & \sin 2\omega_\oplus T_\oplus \Big\{ \beta\Big[-\tfrac{3}{4}s_\xi \tilde{d}_-\Big]\Big\} \\
+\ & \Big\{ \beta\Big[-\tfrac{1}{4}s_\xi \tilde{d}_{XY} + \tfrac{1}{4}s_\xi \tilde{g}_{ZX} + \tfrac{1}{4}s_\xi \tilde{g}_{ZY} + \tfrac{1}{2}s_\xi \tilde{H}_{ZT}\Big] + \beta^2\Big[\tfrac{1}{2}c_\xi \tilde{d}_Z\Big]\Big\} \quad,
\end{aligned} \tag{A.2}
$$

$$\tilde{g}_{\bar{d}} - \tilde{g}_d = \cos\omega_\oplus T_\oplus \left\{\beta\left[2c_\xi\tilde{g}_{XY}\right] + \beta^2\left[\tfrac{1}{2}s_\xi\tilde{b}_X + s_\xi\tilde{g}_{DX} - s_\xi\tilde{g}_{TX}\right]\right\}$$
$$+ \sin\omega_\oplus T_\oplus \left\{\beta\left[2c_\xi\tilde{g}_{YX}\right] + \beta^2\left[\tfrac{1}{2}s_\xi\tilde{b}_Y + s_\xi\tilde{g}_{DY} + s_\xi\tilde{g}_{TY}\right]\right\}$$
$$+ \cos 2\omega_\oplus T_\oplus \left\{\beta\left[s_\xi\tilde{g}_{ZX} - s_\xi\tilde{g}_{ZY}\right] + \beta^2\left[c_\xi\tilde{g}_{TZ}\right]\right\}$$
$$+ \sin 2\omega_\oplus T_\oplus \left\{\beta\left[s_\xi\tilde{b}_T - 2s_\xi\tilde{g}_c - s_\xi\tilde{g}_T\right] + \beta^2\left[-c_\xi\tilde{g}_-\right]\right\}$$
$$+ \left\{\beta\left[-s_\xi\tilde{g}_{ZX} - s_\xi\tilde{g}_{ZY}\right] + \beta^2\left[\tfrac{1}{2}c_\xi\tilde{b}_Z + c_\xi\tilde{g}_{DZ}\right]\right\}\quad, \tag{A.3}$$

$$\tilde{c}_{\bar{q}} - \tilde{c}_q = \cos\omega_\oplus T_\oplus \left\{\beta\left[\tilde{c}_{TY}\right]\right\} + \sin\omega_\oplus T_\oplus \left\{\beta\left[-\tilde{c}_{TX}\right]\right\}$$
$$+ \cos 2\omega_\oplus T_\oplus \left\{\beta^2\left[-\tfrac{1}{2}\tilde{c}_-\right]\right\} + \sin 2\omega_\oplus T_\oplus \left\{\beta^2\left[-\tfrac{1}{2}\tilde{c}_Z\right]\right\} + \left\{\beta^2\left[\tfrac{1}{6}\tilde{c}_Q\right]\right\}\quad,\quad\text{and} \tag{A.4}$$

$$\tilde{g}_{\bar{q}} - \tilde{g}_q = \cos\omega_\oplus T_\oplus \left\{\beta\left[\left(\tfrac{1}{2} - \tfrac{3}{2}c_{2\xi}\right)\tilde{g}_{YX} + \left(\tfrac{1}{2} + \tfrac{3}{2}c_{2\xi}\right)\tilde{g}_{YZ}\right] + \beta^2\left[-\tfrac{3}{4}s_{2\xi}\tilde{g}_{TY}\right]\right\}$$
$$+ \sin\omega_\oplus T_\oplus \left\{\beta\left[\left(-\tfrac{1}{2} + \tfrac{3}{2}c_{2\xi}\right)\tilde{g}_{XY} + \left(-\tfrac{1}{2} - \tfrac{3}{2}c_{2\xi}\right)\tilde{g}_{XZ}\right] + \beta^2\left[\tfrac{3}{4}s_{2\xi}\tilde{g}_{TX}\right]\right\}$$
$$+ \cos 2\omega_\oplus T_\oplus \left\{\beta\left[\tfrac{3}{4}s_{2\xi}\tilde{b}_T + \tfrac{3}{2}s_{2\xi}\tilde{g}_c + \tfrac{3}{4}s_{2\xi}\tilde{g}_T\right] + \beta^2\left[\left(-\tfrac{3}{8} + \tfrac{3}{8}c_{2\xi}\right)\tilde{g}_-\right]\right\}$$
$$+ \sin 2\omega_\oplus T_\oplus \left\{\beta\left[\tfrac{3}{4}s_{2\xi}\tilde{g}_{ZX} - \tfrac{3}{4}s_{2\xi}\tilde{g}_{ZY}\right] + \beta^2\left[\left(-\tfrac{3}{8} + \tfrac{3}{8}c_{2\xi}\right)\tilde{g}_{TZ}\right]\right\}$$
$$+ \left\{\beta\left[-\tfrac{3}{4}s_{2\xi}\tilde{b}_T + \tfrac{3}{4}s_{2\xi}\tilde{g}_T\right] + \beta^2\left[\left(\tfrac{1}{8} + \tfrac{3}{8}c_{2\xi}\right)\tilde{g}_Q\right]\right\}. \tag{A.5}$$

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[32] Actually, they are related by a succession of two transformations: a particle transformation to boost the atom followed by an observer transformation to measure the atom in its rest frame. Since the SME preserves observer transformations, the particle transformation is the relevant part for this discussion.

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[34] Of course, since detailed timing data was not kept, it is possible that the sines and cosines actually average to something appreciably different from zero. This could lead to bounds on an expanded set of coefficient combinations. In this work, we restrict attention to sensitivity that is known with certainty, which corresponds to setting the averages to zero.